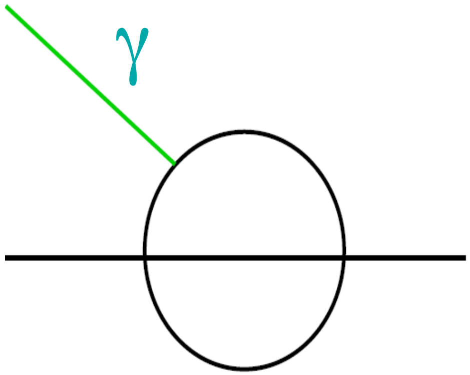
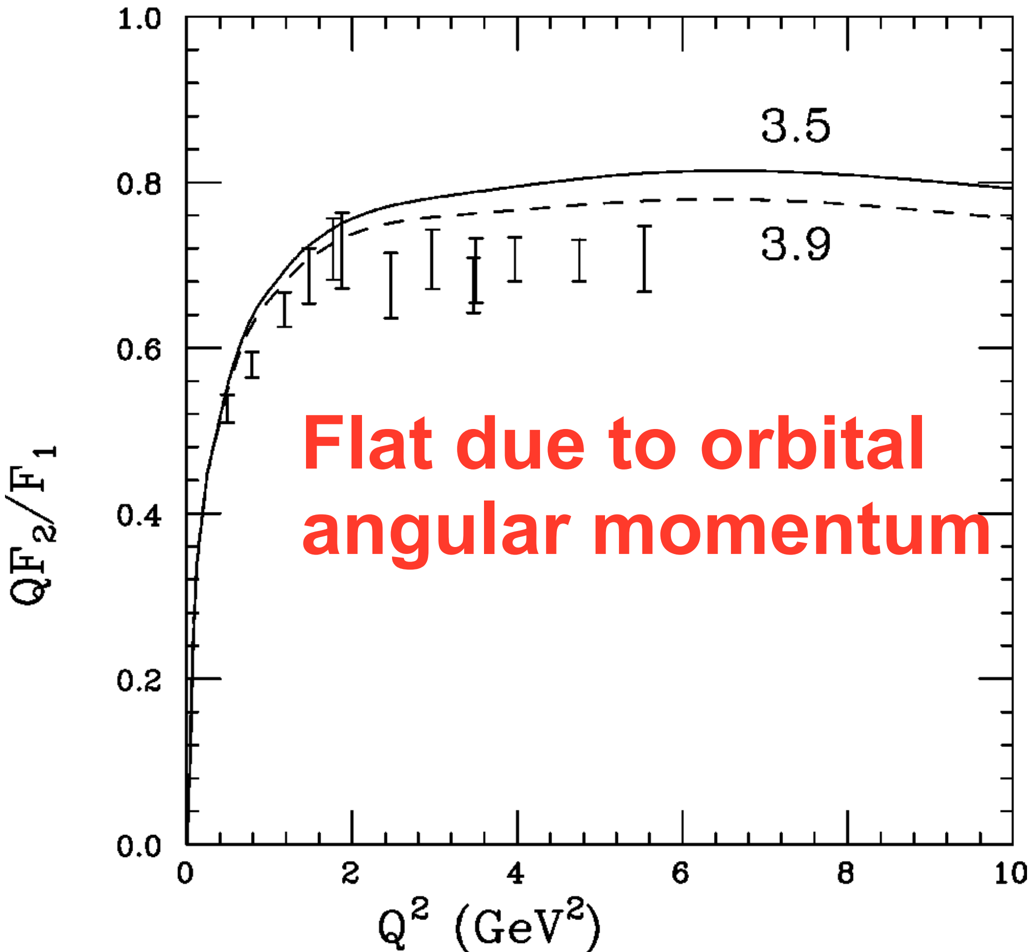
Spin and the Proton Transverse Shape

- **Proton form factor, model calculation-proton not round via spin dependent density**
- **Model independent neutron charge density**
- **Measure shape of proton on lattice (impact parameter dependent GPD) coordinate-space probability, and in experiment (TMD): TMD is momentum-space probability**
- **GAM "Transverse Charge Densities"**

Ratio of Pauli to Dirac Form Factors 1995 Frank,Jennings, Miller theory, data 2000

Impulse approximation





Model proton wave function $\Psi(\mathbf{k}_\perp, \mathbf{K}_\perp, \xi, \eta)$

Poincare invariant

Light front variables for boost: $\mathbf{K} \to \mathbf{K} + \eta \mathbf{q}_\perp$

Dirac spinors carry orbital angular momentum

Model exists

- lower components of Dirac spinor
- orbital angular momentum
- shape of proton?? Wigner Eckart

  no quadrupole moment
- spin dependent densities SDD

  non-relativistic example

I: Non-Rel. $p_{1/2}$ proton outside 0^+ core

$$\langle \mathbf{r}_p | \psi_{1,1/2s} \rangle = R(r_p) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}}_p | s \rangle$$

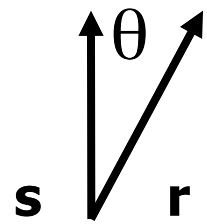
Binding pot'l rotationally invariant

$$\rho(r) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) | \psi_{1,1/2s} \rangle = R^2(r)$$

probability proton at $\mathbf{r}$ & spin direction $\mathbf{n}$:

$$\rho(\mathbf{r}, \mathbf{n}) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) \frac{(1 + \boldsymbol{\sigma} \cdot \mathbf{n})}{2} | \psi_{1,1/2s} \rangle$$

$$= \frac{R^2(r)}{2} \langle s | \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} (1 + \boldsymbol{\sigma} \cdot \mathbf{n}) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} | s \rangle$$



$\mathbf{n} \parallel \hat{\mathbf{s}}$: $\qquad \rho(\mathbf{r}, \mathbf{n} = \hat{\mathbf{s}}) = R^2(r) \cos^2 \theta$

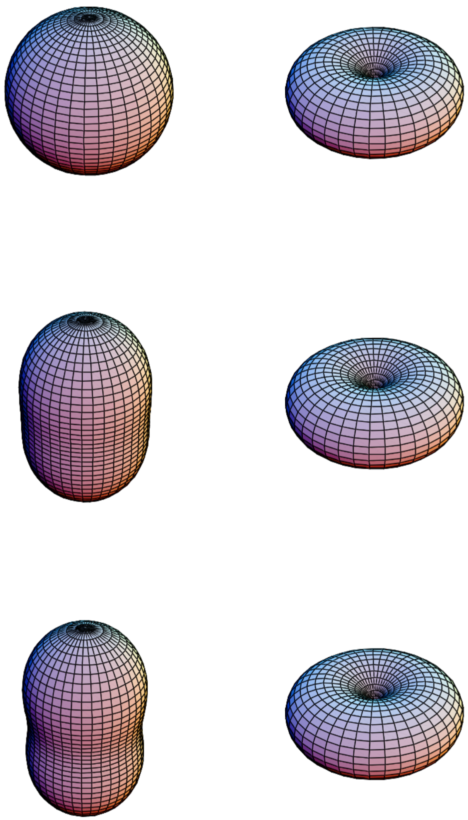
$\mathbf{n} \parallel -\hat{\mathbf{s}}$: $\qquad \rho(\mathbf{r}, \mathbf{n} = -\hat{\mathbf{s}}) = R^2(r) \sin^2 \theta$

non-spherical shape depends on spin direction

Shapes of the proton

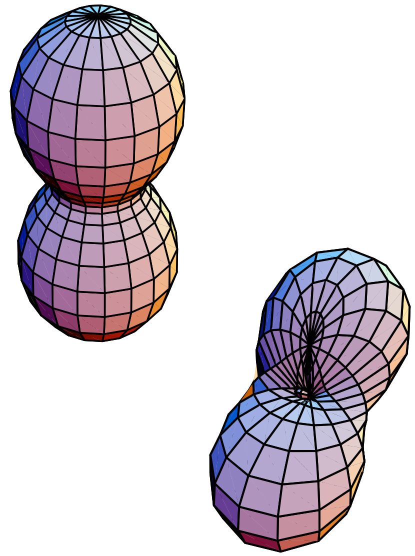
PRC69,022201

Momentum space



vectors n, K, S

Coordinate space



Pretzelocity

How to measure?-Lattice and/or experiment

Relation between coordinate and momentum space densities? Model independent technique needed.

Model Independent Technique

- Light front coordinates, ∞ momentum frame, IMF

$$\text{“Time”}\ x^+ = (ct+z)/\sqrt{2} = (x^0+x^3)/\sqrt{2},\ \text{“evolution”}\ \mathrm{p}^- = (\mathrm{p}^0-\mathrm{p}^3)/\sqrt{2}$$

$$\text{“Space”}\ x^- = (x^0-x^3)/\sqrt{2},\ \text{“Momentum”}\ p^+ = (p^0+p^3)/\sqrt{2}$$

$$\text{“Transverse position, momentum, } \mathbf{b}, \mathbf{p}$$

These coordinates are used to analyze form factors, deep inelastic scattering, GPDs,TMDS

Model independent transverse charge density

$$J^+(x^-, \mathbf{b}) = \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b)$$

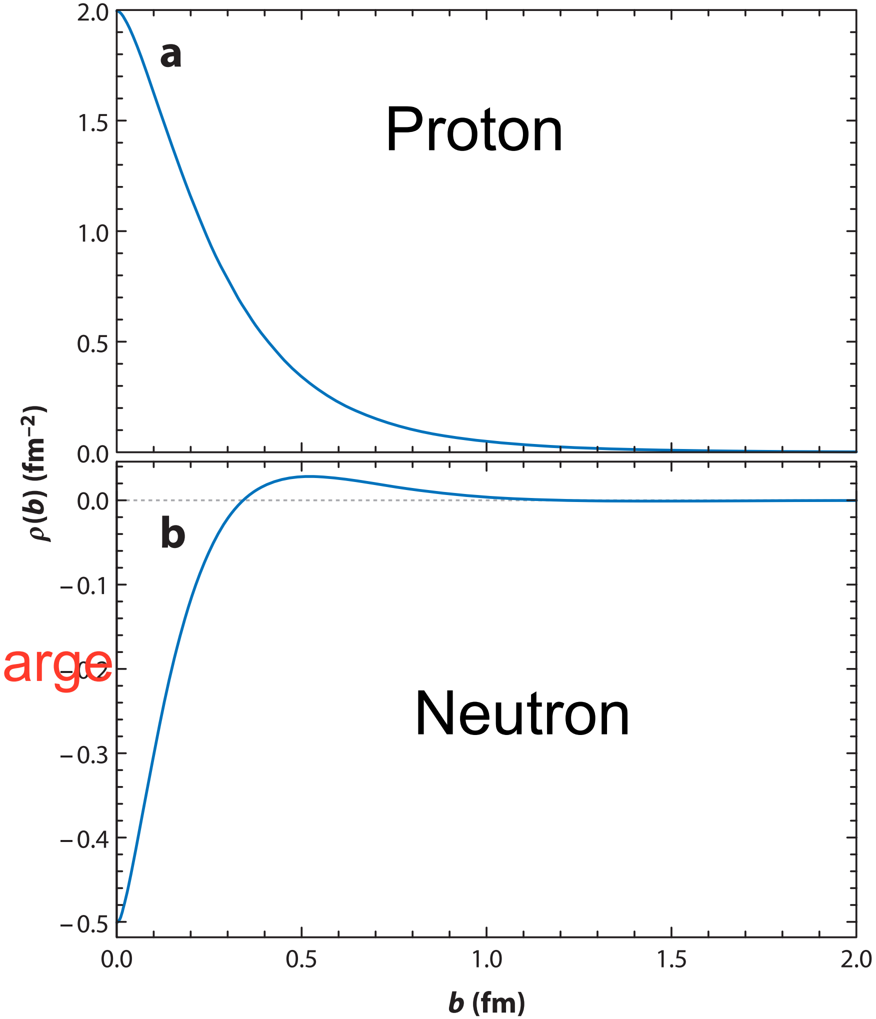
Charge Density operator IMF

$$\rho_\infty(x^-, \mathbf{b}) = \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle$$

$$F_1 = \langle p^+, \mathbf{p}', \lambda | J^+(0) | p^+, \mathbf{p}, \lambda \rangle$$

$$\boxed{\rho(b) \equiv \int dx^- \rho_\infty(x^-, \mathbf{b}) = \int \frac{QdQ}{2\pi} F_1(Q^2) J_0(Qb)}$$

Transverse charge densities from parameterizations (Alberico)



Negative central charge density

Negative central density - GAM PRL '07

Generalized Coordinate Space Densities

$$\rho^{\Gamma}(\mathbf{b}) = \sum_q e_q \int dx^- q_+(x^-, \mathbf{b}) \gamma^+ \Gamma q_+(x^-, \mathbf{b})$$

$$\Gamma = 1/2(1 + \mathbf{n} \cdot \gamma) \text{ gives spin-dep density}$$

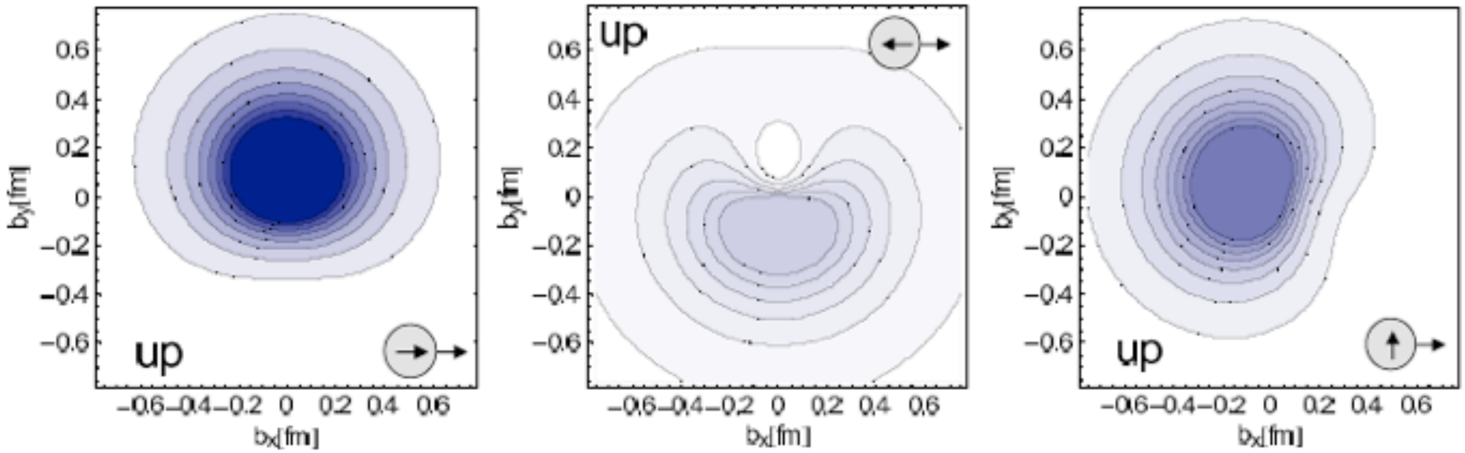
Local operators calculable as x moments on lattice M. Göckeler et al PRL98,222001

$$\widetilde{A}''_{T10} \sim \text{sdd}$$

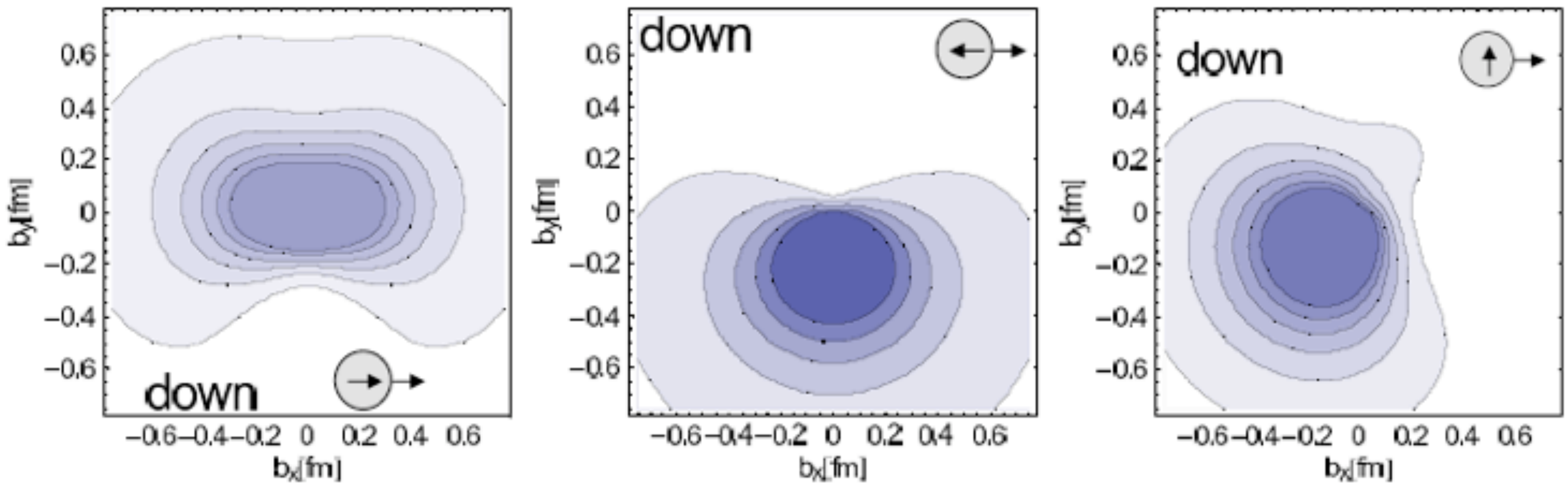
spin-dependent density

Schierholtz, Zanotti 2009 -this quantity is not zero, proton is not round

Spin dependent densities-transverse-Lattice QCDSF, Zanotti, Schierholz...

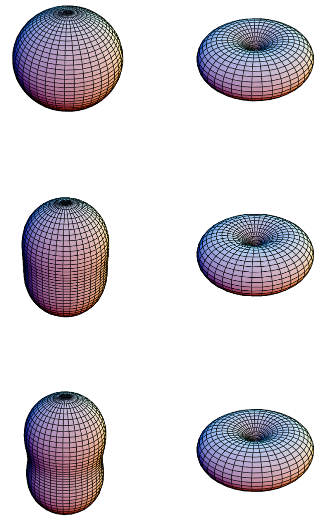


This is not zero! proton is not round



Shapes of the proton

- Relate spin dependent density to experiment
- **Phys.Rev.C76:065209,2007**



Field-theoretic spin dependent momentum density is related to the transverse momentum distribution $h_{1T}^{\perp}$

$$\Phi^{[\Gamma]}(x, \mathbf{K}_T) = \int \frac{d\xi^- d^2\xi_T}{2\,(2\pi)^3}\, e^{iK\cdot\xi}\, \langle P, S|\overline{\psi}(0)\,\Gamma\,\mathcal{L}(0,\xi;n_-)\,\psi(\xi)|P,S\rangle\Big|_{\xi^+=0}$$

Mulders Tangerman'96

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{K}_T) = S_T^i\, h_1(x, K_T^2) + \frac{\left(K_T^i K_T^j - \frac{1}{2}K_T^2\delta_{ij}\right) S_T^j}{M^2}\, h_{1T}^{\perp}(x, K_T^2)$$

$$\sigma^{i+}\gamma^5 \sim \gamma^0\gamma^+\sigma^i,$$

then relate equal time to $\xi^+ = 0$ by integration over x

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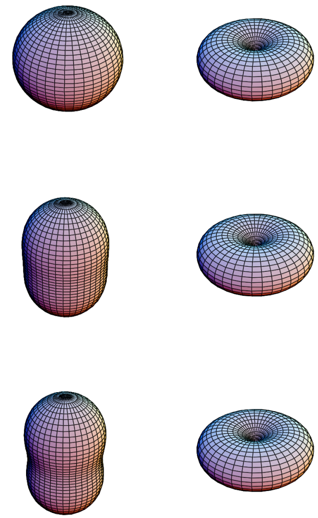

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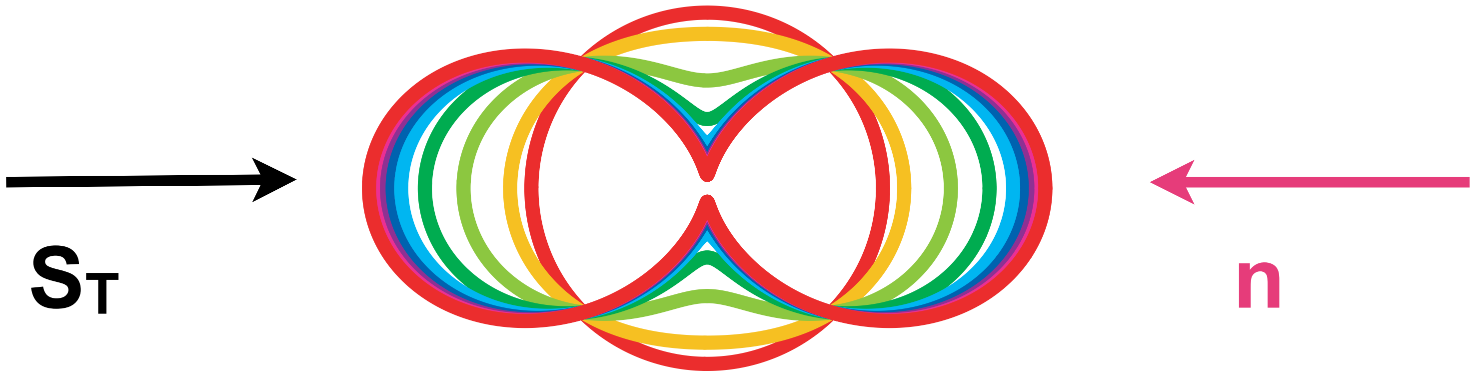
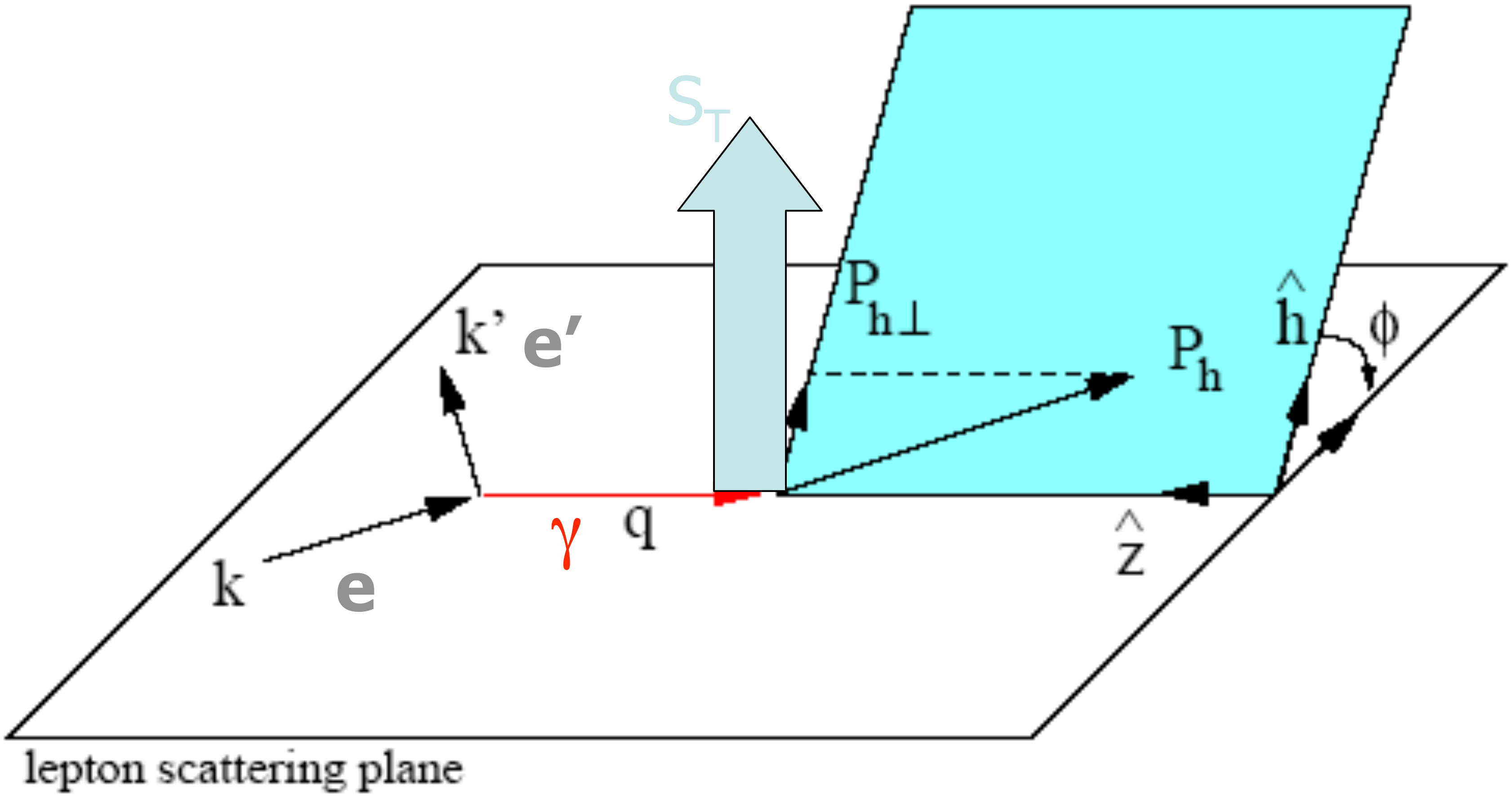
Transverse Shapes of the Proton



Measure $h_{1T}^{\perp}$:e, ↑ p → e’, π X

H. Avakian LOI at Jlab



Cross section has term proportional to cos 3ϕ

Boer Mulders ’98 there are other ways to see $h_{1T}^{\perp}$

Summary

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The Proton